Test 1 - MTH 1400 Online

Dr. Adam Graham-Squire, Summer 2019

Name: Key

I pledge that I have neither given nor received any unauthorized assistance on this exam.

____
(*signature*)

43 min.

DIRECTIONS

1. Don't panic.
2. You should set up Honorlock proctoring now, if you have not done so already.
3. Show all of your work and use correct notation! A correct answer with insufficient work or incorrect notation will lose points.
4. Clearly indicate your answer by putting a box around it.
5. Cell phones are not allowed on this test. No calculators are allowed on the first 7 questions of the test. Calculators ARE allowed on the last 5 questions, however you should still show all of your work. You will initially receive the entire test, and you will NOT be allowed a calculator. Once you have finished everything you can without a calculator, you should turn in the first part of the test (????????) to the proctor. The proctor can then give you your calculator and you can finish the remaining questions. You are not allowed to go back to the No Calculator portion once you have been given your calculator.
6. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
7. If you need it, the quadratic formula is $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
8. Make sure you sign the pledge.
9. Number of questions = 13. Total Points = 65.

1. (5 points) Solve the equation

$$\log_2 3x - \log_2 6 = 5.$$

Hint: Use logarithm laws to combine the left-hand side of the equation into a single logarithm, then rewrite the logarithmic equation into exponential form and solve.

$$\log_2\left(\frac{3x}{6}\right) = 5 \quad \checkmark$$

$$\Rightarrow \quad \log_2\left(\frac{x}{2}\right) = 5 \quad \checkmark$$

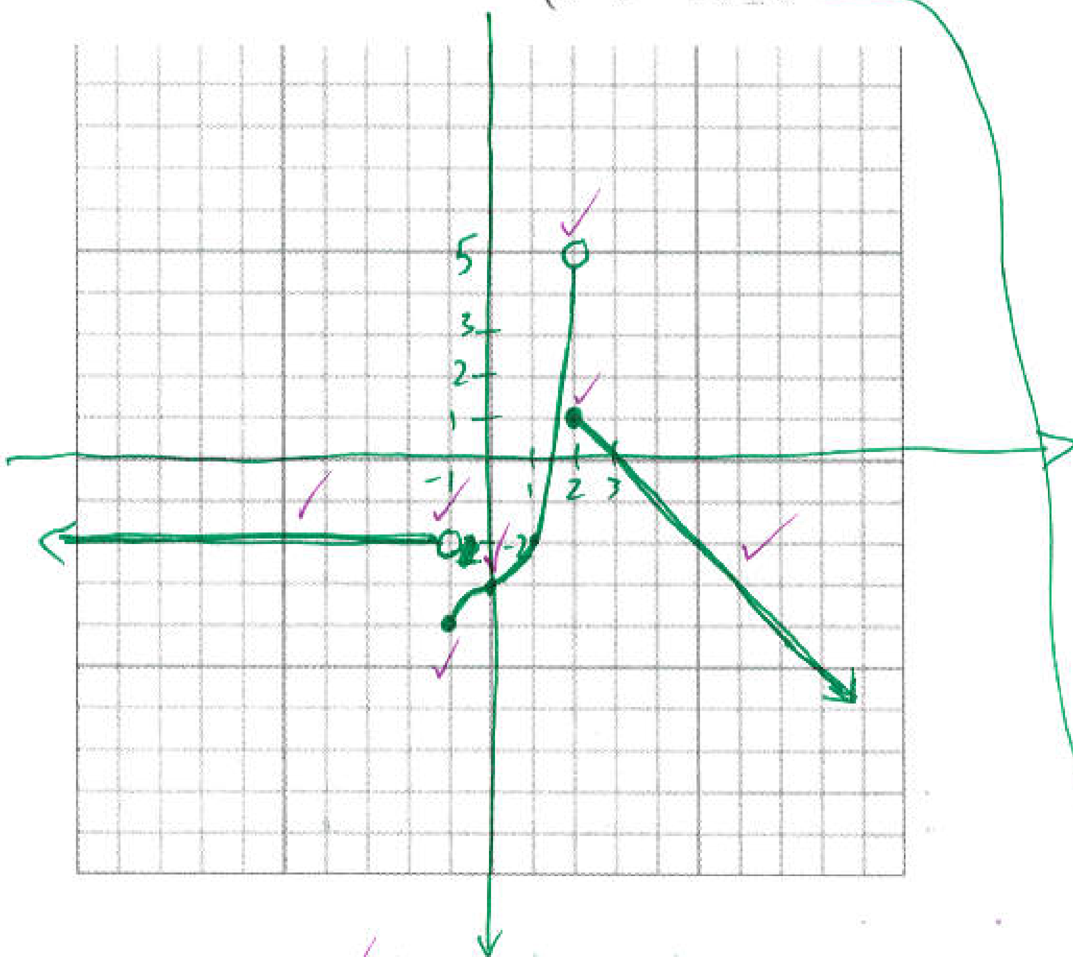
$$\Rightarrow \quad \frac{x}{2} = 2^5 \quad \checkmark$$

$$\Rightarrow \quad x = 2 \cdot (2^5) \quad \checkmark$$

$$\boxed{x = 64} \quad \checkmark$$

2. (5 points) Use the graph paper given below to sketch a graph of the piecewise function. Make sure to label your graph with x- and y-axes, numbers, and open or closed circles if appropriate.

$$f(x) = \begin{cases} -2 & \text{if } x < -1 \\ x^3 - 3 & \text{if } -1 \le x < 2 \\ 3 - x & \text{if } x \ge 2 \end{cases}$$

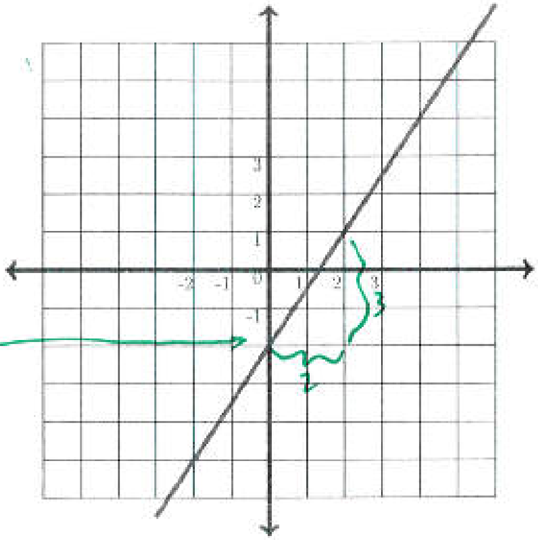


x	y
-1	-4
0	-3
1	-2
2	5

x	y
2	1
3	0
4	-1

3. (a) (2 points) For the graphed line, write the equation for the line in the form $y = mx + b$.

-2



y-intercept $= -2$

rise $= 3$
run $= 2$ } → slope $= \frac{3}{2}$

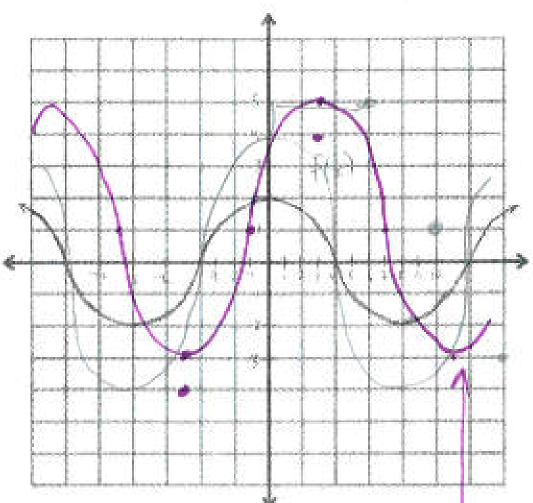
$$\boxed{y = \frac{3}{2} - 2}$$

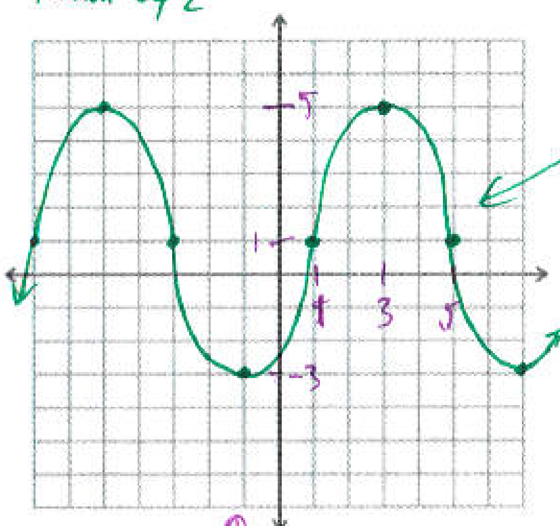
(b) (3 points) Given the graph of $f(x)$ below, stretch, shift and/or flip the graph to correctly sketch the graph of $y = 2f(x-3)+1$. You can graph $y = 2f(x-3)+1$ on the same graph as f, or on the blank graph next to it, whatever you prefer. Make sure to clearly label what your answer is.

shift up 1

shift right by 3

stretch up/down by 2





answer.

This or this

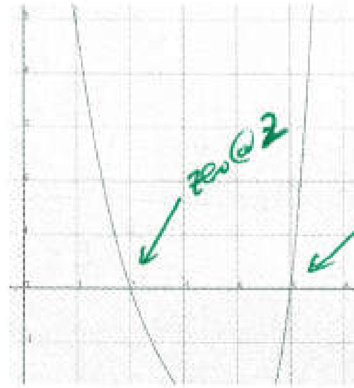
4. (5 points) We wish to find all zeroes of the polynomial $f(x) = x^5 - 11x^4 + 39x^3 - 21x^2 - 172x + 260$. We know the following facts:

- $f(x)$ factors as $f(x) = (x^2 - 4)(x^3 - 11x^2 + 43x - 65)$.

$(x-2)(x+2)$

Zeroes $= 2, -2$

- This is a small chunk of the graph of $f(x)$:



zero @ 5

means

$(x-2)$

$(x-5)$

are factors

Use those facts to find all zeroes of $f(x)$, both real and complex.

$$\begin{array}{r|l} & x^2 - 6x + 13 \\ x-5 & x^3 - 11x^2 + 43x - 65 \\ & -(x^3 + 5x^2) \\ \hline & -6x^2 + 43x \\ & -(+6x^2 + 30x) \\ \hline & 13x - 65 \\ & 13x - 65 \\ \hline & 0 \end{array}$$

$x^2 - 6x + 13$ $a = 1$ $b = -6$ $c = 13$

$$x = \frac{6 \pm \sqrt{36 - 4(1)(13)}}{2}$$

$$= \frac{6 \pm \sqrt{36 - 52}}{2}$$

$$= \frac{6 \pm \sqrt{-16}}{2}$$

$$= \frac{6}{2} \pm \frac{4i}{2}$$

$$= 3 \pm 2i$$

Zeroes are:

$$x = \boxed{2, -2, 5, 3+2i, 3-2i}$$

5. (5 points) Find all vertical and horizontal asymptotes of $y = \dfrac{2x^2+13x-7}{5x+3}$, if any exist. If a vertical or horizontal asymptote does NOT exist, explain why. Explain/show your work.

$$y = \frac{(2x-1)(x+7)}{5x+3}$$

vertical asymptote if $5x+3=0$

$\Rightarrow \boxed{x = \frac{-3}{5}}$

horizontal asymptote? As $x \to \infty / -\infty$

Since top power is bigger, the function goes to $\infty / -\infty$

$\boxed{\text{so no horizontal asymptote}}$

7. (5 points) Let $f(x) = \dfrac{3}{x+5}$ and $g(x) = 4x + 1$.

(a) Calculate the composition of functions $(f \circ g)(x)$. Simplify, if possible.

(b) Calculate the composition of functions $(g \circ f)(x)$. Simplify, if possible.

(c) Calculate the composition of functions $(g \circ g)(x)$. Simplify, if possible.

(a) $f(g(x)) = \dfrac{3}{(4x+1)+5} = \boxed{\dfrac{3}{4x+6}}$ 1.5

(b) $g(f(x)) = 4\left(\dfrac{3}{x+5}\right) + 1 = \boxed{\dfrac{12}{x+5} + 1}$ or $\dfrac{17+x}{x+5}$ 2

(c) $g(g(x)) = 4(4x+1) + 1 = 16x + 4 + 1 = \boxed{16x+5}$ 1.

8. (5 points) The approximate acreage of soybeans planted in the US can be modeled by the function

$$S(t) = \frac{124}{3 + 59e^{-0.02t}}$$

where $t = 0$ corresponds to the year 1900, and S(t) is measured in *millions* of acres of soybeans planted. Use this function to answer the questions below.

(a) How many acres of soybeans were planted in the US in the year 1900 ($t = 0$)?

(b) How many acres were planted in 2019?

(c) The model assumes that the acres planted will stabilize as time goes on (that is, as t goes to infinity). What is that (maximum) stable acreage S approaches?

(a) $S(0) = \frac{124}{3+59e^0} = \frac{124}{62} = \boxed{2 \text{ million acres in } 1900}$

1.5

(b) $2019 \Rightarrow t = 119, \quad S(119) = \frac{124}{3+59e^{-0.02(119)}}$

$= \frac{124}{3+5.46} = \boxed{14.65 \text{ million acres}}$

1.5

(c) As $t \to \infty$, $e^{-0.02t} \Rightarrow e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$

2

So $S(t) \Rightarrow \frac{124}{3+59(0)} = \frac{124}{3} = \boxed{41.33 \text{ million acres}}$

−0.5 if no millions

9. (5 points) Teresa's taco truck currently sells 2,500 tacos per day, for \$6 each. Teresa estimates that, for each \$1 she increases the price of a taco, she will sell 300 less tacos per day (fewer people will want to buy if the price increases).

(a) How much money does Teresa currently make (total) per day? How much revenue would she make if she increased the price of a taco to \$7 and she sold 300 fewer tacos?

(b) Find a function that models Teresa's revenue per day in terms of the price of a taco.

(c) Find the taco price and total revenue that maximizes revenue per day from taco sales. Round to nearest cent

⇒

(a) $2500 \cdot \$6 = \boxed{\$15{,}000}$ ✓

at \$7, she will sell only 2200 tacos

$\Rightarrow \$7 \cdot 2200 = \boxed{\$15{,}400}$ ✓✓

(b) Let x be the # of dollars more than 6 that Teresa charges. Then ✓

$$\text{Revenue} = \underbrace{(6+x)}_{\text{Price per taco}}\underbrace{(2500-300x)}_{\text{\# sold}}$$ ✓✓

$$R(x) = 15000 + 700x - 300x^2$$

Max at $x = \frac{-b}{2a} = \frac{-700}{-600} = 1.1\overline{6}$ ✓ ✓

(c)

$\Rightarrow \boxed{\text{Price} = \$7.17}$ ✓

$\text{Revenue} = R(1.17) = (7.17)(2500 - 300(1.17)) = \boxed{\$15408.33}$ ✓

10. (2 points) Find all solutions to the equation

$$2500 = 400e^{0.37x}.$$

Round answer(s) to nearest 0.01.

$$\frac{2500}{400} = e^{0.37x}$$

$$\ln\left(\frac{2500}{400}\right) = 0.37x$$

$$\frac{\ln\left(\frac{25}{4}\right)}{0.37} = x$$

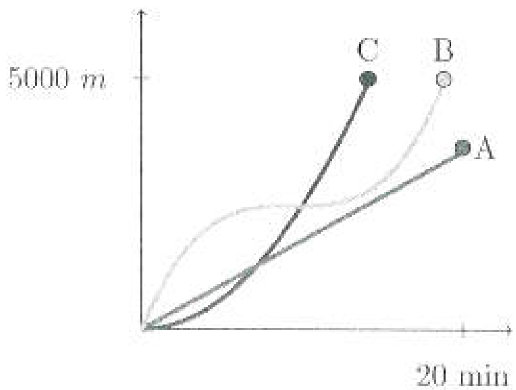
$$\boxed{4.95 = x}$$

11. (3 points) Three runners compete in a 5000-meter race, which lasted only 20 minutes. The graph shows the distance run (in meters) as a function of time (in minutes) for each runner. Describe in words what the graph tells you about this race. In particular, answer these questions, and explain how you know:

(a) Who won the race? Did each runner finish the race?

(b) What happened to Runner B?

(c) Was the winner of the race in the lead the whole time? If not, how many times did the lead change hands?



(a) C won the race, because they reach 5000m at the earliest time. A did not finish the race, because they were not up to 5000m when they hit 20 min.

(b) Runner B started out fast (in the lead), but got tired or fell and did not go very far in the middle of the race. She picked up speed at the end and finished the race.

(c) No. B initially led, and then C overtook them about midway through. The lead changed hands once.

12. (10 points) Jack wants to build a rectangular pen in which to keep some of his animals. He will build the pen next to a pond so that the animals can drink water, and he won't need to put fencing on the pond side of the pen. Jack needs to keep the wombats, capybara, and sloths separate, though, so he decides to put two lines of widthwise–with the fencing around the outside, this will create 3 rectangular pens enclosed by the fence but open to the pond.

(a) Draw a diagram of the situation. Let x be the length of the pen and y be the width.

(b) If Jack wants the pen to be $x = 45$ feet long and $y = 20$ feet wide on the outside (so each individual pen would be 15 by 20), how much fencing does he need total? Note that you need to include the fencing that goes across the inside as well as the fencing on the outside.

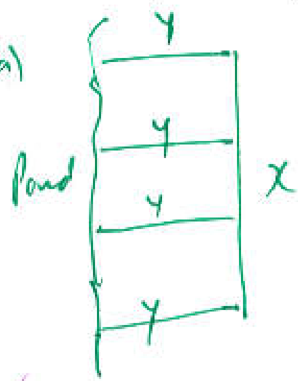
(c) Write a general expression for the total amount of fencing needed in terms of the variables x and y.

(d) If Jack had 1200 total feet of fencing, and he wanted the pen to be 250 feet long on the outside, how wide could he make the pen? (Note: it might help to use your answer from (c))

(e) Assuming that Jack has 1200 feet of fencing, the general expression for the total *area* of the pen in terms of the variable x is given by $A(x) = 300x - \frac{x^2}{4}$. Explain why that equation makes sense (that is, explain where it comes from).

(f) Use the equation from (e) to find the dimensions of the pen that would give a *maximum* area. Show your work (or explain how you came up with your answer) and explain how you know it is a maximum.

(a)



✓

(b) Needs $4(20) + 45 = \boxed{125 \text{ feet}}$ 1.5

(c) fencing $= 4(y) + x = \boxed{4y + x}$ 1.5

(d) $1200 = 4(y) + 250$ 1.5

$\Rightarrow 950 = 4y \Rightarrow y = \frac{950}{4} = \boxed{237.5 \text{ ft}}$

(e) $1200 = 4y + x \Rightarrow y = \frac{1200 - x}{4}$

2

$\text{Area} = x \cdot y = x\left(\frac{1200 - x}{4}\right) = x\left(300 - \frac{x}{4}\right) = 300x - \frac{x^2}{4}$

(f) Graphing $f(x) = 300x - \frac{x^2}{4} = x\left(300 - \frac{x}{4}\right)$

2.5

Max at $x = \frac{-b}{2a} = \frac{-300}{2(\frac{-1}{4})}$ $x = 0$ $x = 1200$

$x = 600 \rightarrow$ ove

13. (5 points) The table below gives the position of a particle moving in a straight line at certain times, where t is time in seconds and p is the position in feet.

$\text{feet/sec} \Rightarrow \dfrac{p_2 - p_1}{t_2 - t_1}$

t	p
2.5	17.625
2.9	25.589
2.95	26.772
2.99	27.751
3	28

Calculate the average rate of change of the particle from

(a) 2.5 to 3 seconds $\Rightarrow \dfrac{28-17.625}{3-2.5} = 20.75$ ft/sec

(b) 2.9 to 3 seconds

(c) 2.95 to 3 seconds $\Rightarrow \dfrac{28-26.772}{0.05} = 24.56$

(d) 2.99 to 3 seconds $\longrightarrow \dfrac{28-27.751}{0.01} = 24.9$

(e) Use your answers from (a) through (d) to estimate how fast (in feet/sec) the particle was moving at exactly 3 seconds.

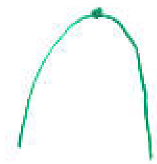
$\hookrightarrow$ a $\to$ d seem to be approaching $\boxed{25 \text{ ft/sec}}$

$\dfrac{28-25.589}{3-2.9} = 24.11$ ft/sec

$20.75 \to 24.11 \to 24.56 \to 24.9 \to ?$

Left blank if you need more room for question 12.

Max b/c $-\frac{x^2}{4}$ means parabola is

opening ~~pointing~~ down $\Rightarrow$ 

$\boxed{\text{Max at } x = 600}$

$\Rightarrow \boxed{y = \frac{600}{4} = 150}$

90000

Move to with calc.

Extra Credit(2 points) Is $2+i$ a zero for the quadratic expression $x^3-5x^2+12x-8$? Substitute into the expression and simplify to check.(Need to show your work)

$$(2+i)^3-5(2+i)^2+12(2+i)-8$$

$$=(4+4i+i^2)(2+i)-5(4+4i+i^2)+24+12i-8$$

$$=\underline{8}+\underline{\underline{8i}}+2i^2+\underline{\underline{4i}}+4i^2+i^3\underline{-20}\underline{\underline{-20i}}-5i^2\underline{+16}\underline{\underline{+12i}}$$

$$=4+4i+i^2+i^3$$

$$i^2=-1$$

$$i^3=i^2\cdot i=-1\cdot i=-i$$

$$=4+4i-1-i$$

$$=3+3i\neq 0$$ so <u>NOT</u> a zero.